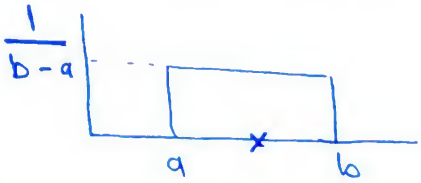
12/12/2015 السبت د. سيد سلام محاضرة [9]

* Continuous Distribution

1] Uniform Distribution

2] Gaussian (Normal) Distribution.

1- uniform distribution



$$A = 1 = K(b-a) = K = \frac{1}{b-a}$$

$$f(x) = \frac{1}{b-a} \qquad a < x < b$$

It is a continuous distribution.

properties of uniform distribution :-

$$\text{mean} = \mu = \frac{a+b}{2}$$

$$\text{Variance} = \frac{(b-a)^2}{12}$$

$$\text{S.D.} = \frac{b-a}{2\sqrt{3}} \Longrightarrow \text{Proof ??}$$

2- The normal distribution (Gaussian distribution)

The normal distribution (Gaussian distribution) or curve is defined as follows :-

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

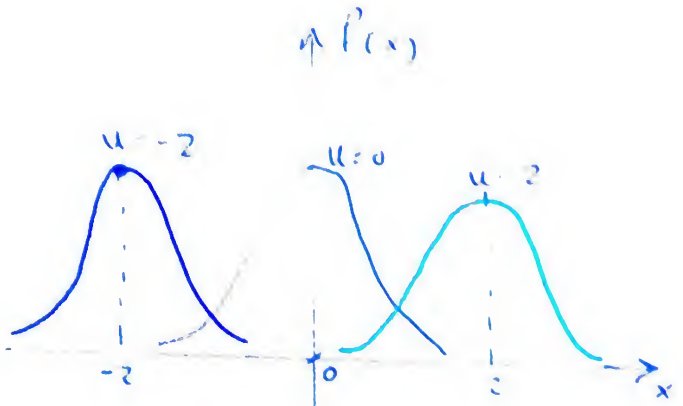
Where m and $\sigma > 0$ are arbitrary constants.

This function is certainly one of the most important examples of a continuous probability distribution.

The two diagrams below show the change

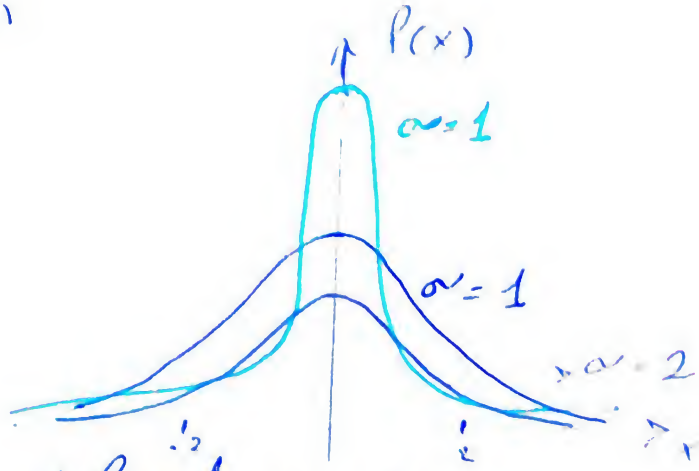
in $f(x)$ as m varies or σ varies.

In particular, observe that these bell shaped curves are symmetric about $x = m$



normal distributions

σ fixed ($\sigma = 1$)



μ fixed ($\mu = 0$)

Properties of normal distribution

mean $= m$

Variance $= \sigma^2$ S.D. $= \sigma$

we denote the normal distribution with mean m and variance σ^2 by $N(m, \sigma^2)$. If we make the substitution

$N \sim (m, \sigma^2)$

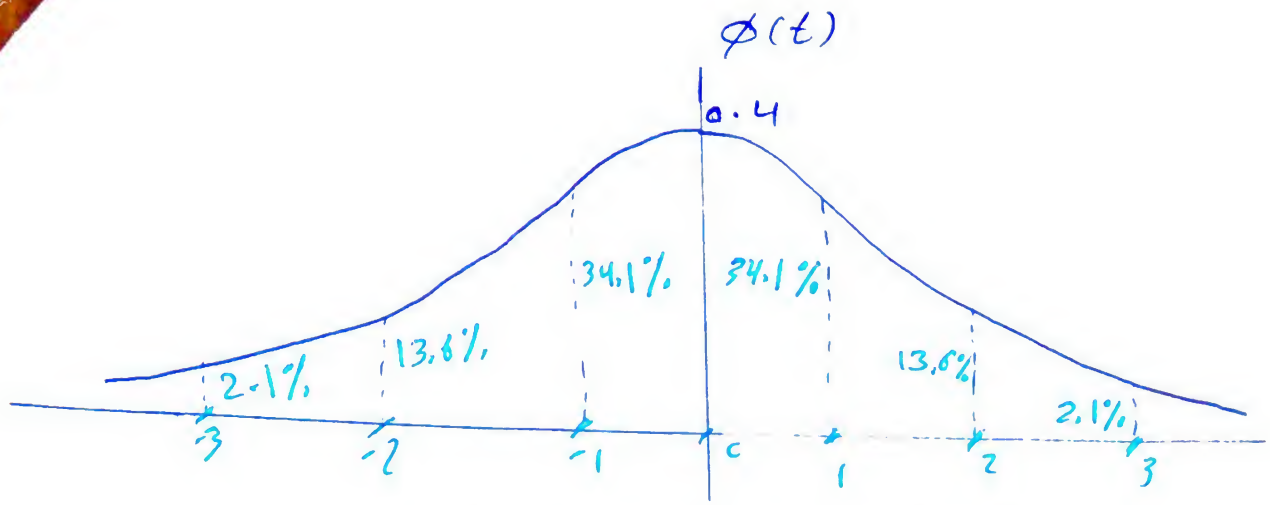
$t = \dfrac{x - m}{\sigma}$ in the above formula (Page 1 [$f(x)$])

for $N(m, \sigma^2)$, we obtain the standard normal (Gaussian) distribution or curve

$$\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2},$$

which has mean $= 0$ and variance $\sigma^2 = 1$.

The graph of this distribution appears below.



normal distribution $N(0, 1)$

We note that for $-1 \leq t \leq 1$ we obtain 68.2% of the area under the curve, and for $-2 \leq t \leq 2$, we obtain 95.4% of the area under the curve. It is tabulated.

A table (1) gives the values of $\phi(t)$ of the Standard normal distribution at $t \geq 0$. Table (2) gives the area under the standard normal curve between $t=0$ and any positive value of t.

The symmetry of the curve about $t=0$ permits us to obtain the area between any two values of t.

table (1) and (2) are uploaded

{ الجداول تكون معطاة في الامتحان }

محتاج لدي الجدول ليه ، تكون محتاج جدول واحد بس (take care)

$\Longrightarrow$ Turn Over

Required :- (about kalman filter)

Report ⟶ اشرح الفلتر، وهات مثال

على System يكون مثل 2nd order أو 3rd order

الفكرة: في ال State space بيكون فيك disturbance وبتحتاج عمل filter

للتخلص منك

$$\dot{x} = Ax + Bu + Gw$$

Gw ← عايز أشيل تأثيره

Now let X be a continuous random variable with normal distribution; we frequently say that x is normally distributed. we compute the probability that X lies between a and b, denoted by $P(a \leq X \leq b)$, as follows:-

- First we change a and b into standard units.

$$a' = \frac{a-m}{\sigma} \text{ and } b' = \frac{b-m}{\sigma} \text{ respectively}$$

- then $P(a \leq X \leq b) = P(a' \leq x \leq b') =$

= the area under the standard normal curve between a' and b'

Here x' is the standerdized random variable corresponding to X and hence X has the standard normal distribution $N(0, 1)$

⟹ Turn Over

Let X be a random variable with the standard normal distribution ϕ. find:

i) $P(0 \leqslant X \leqslant 1.42$

ii) $P(-0.73 \leqslant X \leqslant 0)$

iii) $P(-1.37 \leqslant X \leqslant 2.01)$

iv) $P(0.65 \leqslant X \leqslant 1.26)$

v) $P(-1.79 \leqslant X \leqslant -0.54)$

vi) $P(X \geqslant 1.13)$

vii) $P(|X| \leqslant 0.5)$

Suppose the weight measurments W of 800 students are normally distributed withe mean 66 Kg and S.D. 5 Kg. find the number N of students with weight:

i) between 65 and 70 kg.

ii) greater than or equal 72 kg.